Reliability Analysis of a Cold Stand By System with Repair-Equipment Failure and Appearance and Disappearance of Repairman with Correlated Life Time

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Abstract - The aim of this paper is to present a reliability analysis of a two unit cold standby system with the assumption that the repair-equipment may also fail during the repair of a failed unit. There is only one repair facility which may be appeared and disappeared randomly. Failure and Repair times of each unit are assumed to be correlated. Using regenerative point technique various reliability characteristics are obtained which are useful to system designers and industrial managers. Graphical behaviors of MTSF and profit function have also been studied.

Keywords - Transition Probabilities, sojourn time, MTSF, Availability, Busy Period, Profit Function, Bivariate exponential distribution.

1. INTRODUCTION

A two identical unit cold stand by systems have widely studied in literature of reliability theory, repair maintenance is one of the most important measures for increasing the reliability of the system. Many authors have studied various system models under different repair policies [1-4] they have assumed that the failure and repair times are uncorrelated random variable. They have also assumed that the repair –equipment used for repairing a failed unit can never failed but in real life this is not so. It is also possible that the repair-equipment may fail for some reason during repair process of a failed unit in this case repairman first repairs the repair-equipment and repairman also need rest so repairman can appear and disappear randomly. Taking this fact into consideration in this paper we investigate a cold stand by unit system model assuming the possibility of failure of repair-equipment during the repair process of a failed unit with appearance and disappearance of repairman with failure and repair of unit as correlated random variables having their joint distribution as bivariate exponential .

2. SYSTEM DESCRIPTION

System consists of two identical units one is operative and other is in cold stand by. There is single repair facility. When repair equipment fails during the repair of any failed unit, repairman starts the repair of failed unit and repair equipment may also fail. The joint distribution of failure and repair times for each unit is taken to be bivariate exponential having density function. Each repaired unit works as good as new.

3. NOTATIONS

For defining the states of the system we assume the following symbols:

A_0:	Unit A is in operative mode
A_{fr}:	Unit A is in failure mode
x:	Constant rate of repair of repair-equipment
θ:	Constant rate of repair-equipment's failure
Ø:	Constant rate disappearance of the repairman the system
ω:	Constant rate appearance of the repairman the system
A_{Fw}:	Unit A in failure mode but in waiting for repairman
X_i(i=1):	Random variables representing the failure times of A unit

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Y_i(i=1): Random variables representing the repair times of A unit

f_i(x,y): Joint pdf of (x_i,y_i);i=1

$$= \alpha_i \beta_i (1-r_i) e^{-\alpha_i x - \beta_i y} I_0(2\sqrt{(\alpha_i \beta_i r_i xy)}); X, Y, \alpha_i, \beta_i > 0;$$
$$0 \le r_i < 1,$$
$$where I_0(2\sqrt{\alpha_i \beta_i r_i xy}) = \sum_{j=0}^{\infty} \frac{(\alpha_i \beta_i r_i xy)^j}{(j!)^2}$$

k_i(Y/X): Conditional pdf of Y_i given X_i=x is given by
$$= \beta_i e^{-\alpha_i r_i x - \beta_i y} I_0(2\sqrt{(\alpha_i \beta_i r_i xy)})$$

g_i(.): Marginal pdf of $X_i = \alpha_i(1-r_i)e^{-\alpha_i(1-r_i)x}$

h_i(.): Marginal pdf of $Y_i = \beta_i(1-r_i)e^{-\beta_i(1-r_i)y}$

$q_{ij}(.), Q_{ij}$ pdf &cdf of transition time from regenerative states pdf &cdf of transition time from regenerative state S_i to S_j.

μ_i : Mean sojourn time in state S_i.

$\oplus$: Symbol of ordinary Convolution
$$A(t) \oplus B(t) = \int_0^t A(t-u)B(u)du$$

 : symbol of stieltjes convolution
$$A(t) \; B(t) = \int_0^t A(t-u)dB(u)$$

3.1 Transition Probability and Sojourn Times

The steady state transition probability can be as follows

$$p_{01} = 1,$$
$$p_{10} = \frac{\beta_1(1-r_1)}{\alpha_1(1-r_1)+\beta_1(1-r_1)+\theta+\phi}$$
$$p_{15} = \frac{\alpha_1(1-r_1)}{\alpha_1(1-r_1)+\beta_1(1-r_1)+\theta+\phi}$$
$$p_{12} = \frac{\theta}{\alpha_1(1-r_1)+\beta_1(1-r_1)+\theta+\phi},$$
$$p_{13} = \frac{\phi}{\alpha_1(1-r_1)+\beta_1(1-r_1)+\theta+\phi},$$
$$p_{21} = \frac{x}{\alpha_1(1-r_1)+x}$$
$$p_{26} = \frac{\alpha_1(1-r_1)}{\alpha_1(1-r_1)+x}$$
$$p_{31} = \frac{\omega}{\alpha_1(1-r_1)+\omega}$$
$$p_{34} = \frac{\alpha_1(1-r_1)}{\alpha_1(1-r_1)+\omega}$$
$$p_{51} = \frac{\beta_1(1-r_1)}{\beta_1(1-r_1)+\theta+\phi}$$
$$p_{54} = \frac{\phi}{\beta_1(1-r_1)+\theta+\phi}$$
$$p_{56} = \frac{\theta}{\beta_1(1-r_1)+\theta+\phi}$$

(1-12)

$$p_{01} = 1$$
$$p_{10} + p_{12} + p_{15} + p_{13} = 1$$
$$p_{10} + p_{12} + p_{13} + p_{14.5} + p_{16.5} + p_{11.5} = 1$$
$$p_{21} + p_{26} = 1$$
$$p_{21} + p_{26.5} = 1$$
$$p_{31} + p_{34} = 1$$
$$p_{51} + p_{54} + p_{56} = 1$$
$$p_{45} = 1$$
$$p_{65} = 1$$
$$\mu_0 = \frac{1}{\alpha_1(1-r_1)},$$
$$\mu_1 = \frac{1}{\beta_1(1-r_1)+\alpha_1(1-r_1)+\theta+\phi},$$
$$\mu_2 = \frac{1}{\alpha_1(1-r_1)+x}, \mu_3 = \frac{1}{\omega+\alpha_1(1-r_1)},$$
$$\mu_4 = \frac{1}{\omega}, \mu_8 = \frac{1}{x}$$

(13-27)

4. ANALYSIS OF CHARACTERISTICS

4.1 MTSF (Mean Time to System Failure)

To determine the MTSF of the system, we regard the failed state of the system as absorbing state, by probabilistic arguments, we get

$$\phi_0(t) = Q_{01} \square \phi_1(t)$$
$$\phi_1(t) = Q_{10} \square \phi_0(t) + Q_{13} \square \phi_3(t) + Q_{12} \square \phi_2(t) + Q_{15}$$
$$\phi_2(t) = Q_{21} \square \phi_1(t) + Q_{26}$$
$$\phi_3(t) = Q_{31}(t) \square \phi_1(t) + Q_{34}$$

(28-31)

Taking Laplace stieltjes transforms of these relations and solving for $\phi_0^{**}(s)$,

$$\phi_0^{**}(s) = \frac{N(s)}{D(s)} \qquad (32)$$

Where

$$N = \mu_0(1 - p_{12}p_{21} - p_{13}p_{31}) + \mu_1 p_{12} + \mu_3 p_{13}$$
$$D = (1 - p_{10} - p_{12}p_{21} - p_{13}p_{31})$$

(33-34)

4.2 Availability Analysis

Let $A_i(t)$ be the probability that the system is in up-state at instant t given that the system entered regenerative state i at t=0.using the arguments of the theory of a regenerative process the point wise availability $A_i(t)$ is seen to satisfy the following recursive relations

$$A_0(t) = M_0(t) + q_{01}(t) \oplus A_1(t)$$
$$A_1(t) = M_1(t) + q_{10}(t) \oplus A_0(t) + q_{13}(t) \oplus A_3(t) + q_{12}(t) \oplus A_2(t) + q_{14.5}(t) \oplus A_4(t)$$
$$+ q_{16.5}(t) \oplus A_5(t) + q_{11.5}(t) \oplus A_1(t)$$
$$A_2(t) = M_2(t) + q_{21}(t) \oplus A_1(t) + q_{25.6}(t) \oplus A_5(t)$$
$$A_3(t) = M_3(t) + q_{31}(t) \oplus A_1(t) + q_{34}(t) \oplus A_4(t)$$
$$A_4(t) = q_{45}(t) \oplus A_5(t)$$
$$A_5(t) = q_{51}(t) \oplus A_1(t) + q_{56}(t) \oplus A_6(t) + q_{54}(t) \oplus A_4(t)$$
$$A_6(t) = q_{65}(t) \oplus A_5(t)$$

(35-42)

Now taking Laplace transform of these equations and solving them for $A_0^*(s)$,

We get

$$A_0^*(s) = \frac{N_1(s)}{D_1(s)} \qquad (43)$$

The steady state availability is $A_0 = \lim_{s \to 0}(sA_0^*(s)) = \frac{N_1}{D_1}$

Where

$$N_1 = \mu_0 p_{12}[-p_{21}(1 - p_{45}p_{54}) - p_{25.6}p_{51}] + \mu_0 p_{13}[-p_{31}(1 - p_{45}p_{54}) - p_{34}p_{45}p_{51}]$$
$$-\mu_0[1 - p_{11.5} - p_{16.5}p_{51}) - p_{45}p_{54}(1 - p_{11.5}) - p_{14.5}p_{51}]$$
$$+\mu_0 p_{56}p_{65}[-p_{31}p_{13} - (1 - p_{11.5}) - p_{12}p_{21}] - p_{01}p_{51}p_{65}[p_{13}\mu_3 + \mu_1 + \mu_2 p_{12}]$$
$$+p_{01}p_{12}\mu_2[(1 - p_{45}p_{54})] + p_{01}\mu_1[(1 - p_{45}p_{54})] + p_{01}p_{13}\mu_3[(1 - p_{45}p_{54})]$$

$$D_1 = \mu_0 p_{10}p_{51} + \mu_1 p_{51} + \mu_2 p_{12}p_{51} + \mu_3 p_{13}p_{51} + \mu_4(1 - p_{10} - p_{11.5} - p_{13}p_{31} - p_{12}p_{21})$$
$$+(\mu_4 p_{54} + \mu_6 p_{56})(1 - p_{12}p_{21} - p_{13}p_{31} - p_{11.5}) + \mu_4(p_{13}p_{34}p_{51} + p_{14.5}p_{51}) - \mu_6(p_{10}p_{56})$$

(44-46)

4.3 Busy Period Analysis of the Repairman

Let $B_i(t)$ be the probability that the repairman is busy at instant t, given that the system entered regenerative state I at t=0.By probabilistic arguments we have the following recursive relations for $B_i(t)$

$$B_0(t) = q_{01}(t) \oplus B_1(t)$$
$$B_1(t) = W_1 + q_{10}(t) \oplus B_0(t) + q_{12}(t) \oplus B_2(t) + q_{13}(t) \oplus B_3(t) + q_{14.5}(t) \oplus B_4(t) +$$
$$q_{16.5}(t) \oplus B_5(t) + q_{11.5}(t) \oplus B_1(t)$$
$$B_2(t) = W_2 + q_{21}(t) \oplus B_1(t) + q_{25.6}(t) \oplus B_5(t)$$
$$B_3(t) = q_{31}(t) \oplus B_1(t) + q_{34}(t) \oplus B_4(t)$$
$$B_4(t) = q_{45}(t) \oplus B_5(t)$$
$$B_5(t) = W_5 + q_{51}(t) \oplus B_1(t) + q_{56}(t) \oplus B_6(t) + q_{54}(t) \oplus B_4(t)$$
$$B_6(t) = W_6 + q_{65}(t) \oplus B_5(t)$$

(47-53)

Taking Laplace transform of the equations of busy period analysis and solving them for $B_0^*(s)$,we get

$$B_0^*(s) = \frac{N_2(s)}{D_1(s)} \qquad (54)$$

In the steady state

$$B_0 = \lim_{s \to 0}(sB_0^*(s)) = \frac{N_2}{D_1} \qquad (55)$$

Where

$$N_2 = \mu_1[p_{01} - p_{01}p_{45}p_{54} - p_{01}p_{56}p_{65}] + \mu_2 p_{01}p_{12}[p_{45}p_{54} - (1 - p_{56}p_{65}]$$
$$+\mu_5[-p_{01}p_{12}p_{25.6} + p_{01}p_{13}p_{34}p_{45} + p_{01}p_{45}p_{14.5} + p_{01}p_{16.5}]$$
$$+\mu_6[-p_{01}p_{12}p_{25.6}p_{56.7}p_{01}p_{13}p_{34}p_{45}p_{56} + p_{01}p_{56}p_{45}p_{14.5} + p_{01}p_{56}p_{16.5}]$$

(56)

D_1 is already specified.

4.4 Expected Number of Visits by the Repairman

We defined as the expected number of visits by the repairman in (0,t],given that the system initially starts from regenerative state S_i

By probabilistic arguments we have the following recursive relations for $V_i(t)$

Taking laplace stieltjes transform of the equations of expected number of visits And solving them for $V_0^{**}(s)$, we get

$$V_0(t) = q_{01}(t) \oplus (1 + V_1(t))$$
$$V_1(t) = q_{10}(t) \oplus V_0(t) + q_{12}(t) \oplus V_2(t) + q_{13}(t) \oplus V_3(t) + q_{14.5}(t) \oplus V_4(t) + q_{16.5}(t) \oplus V_5(t) + q_{11.5}(t) \oplus V_1(t)$$
$$V_2(t) = q_{21}(t) \oplus V_1(t) + q_{25.6}(t) \oplus V_5(t)$$
$$V_3(t) = q_{31}(t) \oplus V_1(t) + q_{34}(t) \oplus V_4(t)$$
$$V_4(t) = q_{45}(t) \oplus V_5(t)$$
$$V_5(t) = q_{51}(t) \oplus V_1(t) + q_{56}(t) \oplus V_6(t) + q_{54}(t) \oplus V_4(t)$$
$$V_6(t) = q_{65}(t) \oplus V_5(t)$$

$$V_0^{**}(s) = \frac{N_3(s)}{D_1(s)} \tag{57-65}$$

In steady state

$$V_0 = \lim_{s \to 0}(sV_0^*(s)) = \frac{N_3}{D_1} \tag{66}$$

Where

$$N_3 = p_{01}p_{12}p_{21}[p_{45}p_{54} - p_{65}p_{56}] - p_{01}p_{12}p_{51}p_{25.6} - p_{01}p_{12}p_{45}p_{25.6} - p_{01}p_{13}p_{45}p_{34}p_{56}p_{65} + p_{01}p_{14.5}p_{45}p_{45}p_{56}p_{65} + p_{01}p_{16.5}p_{45}p_{54} \tag{67}$$

D_1 is already specified

5. PROFIT ANALYSIS

The expected total profit incurred to the system in steady state is given by

$$P = C_0A_0 - C_1B_0 - C_2V_0 \tag{68}$$

Where

C_0 =revenue/unit uptime of the system

C_1 =cost/unit time for which repairman is busy

C_2 =cost/visit for the repairman

6. CONCLUSION For a more clear view of the system characteristics w.r.t. the various parameters involved, we plot curves for MTSF and profit function in figure-2 and figure-3 w.r.t the failure parameter (α_1) of unit A for three different values of correlation coefficient (r_{11} =0.25, r_{12} =0.50, r_{13} =0.75), between X_1 and Y1 while the other parameters are kept fixed as

$$\phi = .005, \beta_1 = .03, \theta = .001, \omega = .04$$
$$x = .05, C_0 = 500, C_1 = 300, C_2 = 50$$

From the figure-2 it is observed that MTSF decreases as failure rate increases irrespective of other parameters. the curves also indicates that for the same value of failure rate,MTSF is higher for higher vaues of correlation coefficient(r),so here we conclude that the high value of r between failure and repair tends to increase the expected life time of the system.

From the figure-3 it is clear that profit decreases linearly as failure rate increases. Also for the fixed value of failure rate, the profit is higher for high correlation (r).

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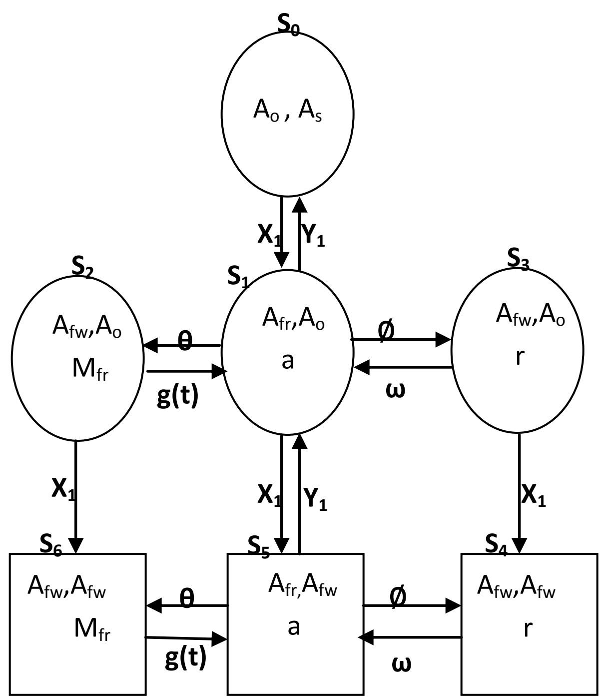
Transition Diagram



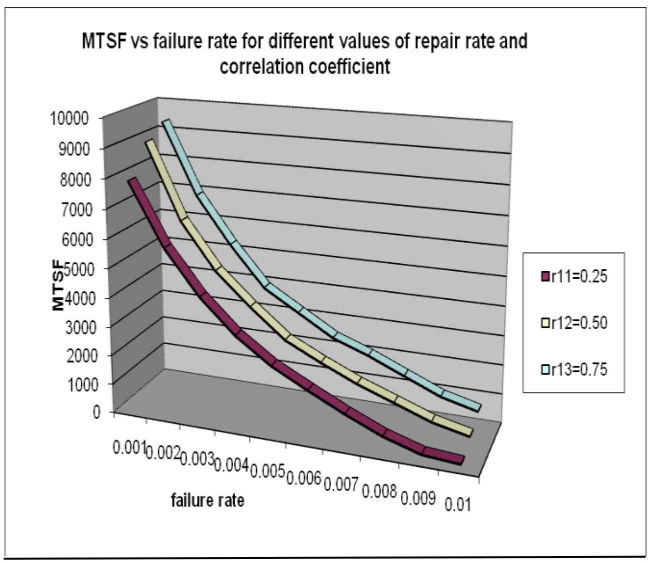


FIGURE -1

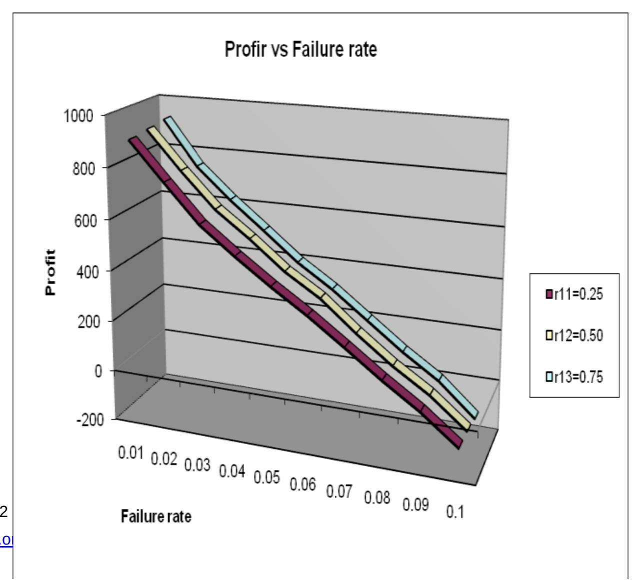


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FIGURE -2